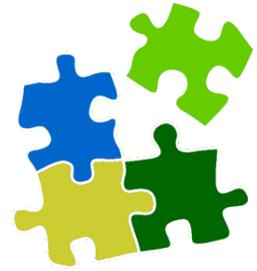


## 2018-19 Puzzle Contests

# Solutions for Contest #2



### Grades 5up Puzzle Solutions:

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1. The witch Margali Szardos tells the truth on all even dates and lies on all odd dates. One day she made exactly four of the following five statements. Which one she did NOT make that day?
- a) I have a prime number of acquaintances
  - b) I have as many acquaintances among witches as among ghosts
  - c) 343 is a perfect cube.
  - d) I always tell the truth.
  - e) I hate exactly three of my acquaintances
- ( 25 pts.)



Answer: The statement c) could not be made.

Solution: Since the statement 343 is a perfect cube , c) is always TRUE. By the problem statement, The witch Margali Szardos lies half the time, d) is always FALSE. So:

- a) I have a prime number of friends. ???
- b) I have as many friends among withches as among ghosts. ???
- c) 343 is a perfect cube.. TRUE ( $343 = 7^3$ )
- d) I always tell the truth. FALSE
- e) I hate exactly three of my acquaintances. ???

Suppose on the day in question Margali Szardos is telling the truth. Then 4 statements must be true. Since we know that d is false, it must be that all other 4 statements are true. But a, b, and c cannot be true at the same time, because a and b together, if both true, imply that he has an even number of acquaintances which is also prime, so he must have exactly 2 acquaintances, while c implies that he has at least three acquaintances. Hence today must be a lying day, in which case she could not make statement c, which is true. We still need to establish that it is possible that a,b and e are simultaneously false (we already know that d is false). A trivial example would be that all her acquaintances are 4 ghosts and she hates all of them.

2. Old Gremlin went to Witch Elvira's Apothecary to buy spider ointment, snake elixir, beladonna extract, and caterpillar juice for his aching bones. He found these remedies on the apothecary shelf in a vile, a glass, a jar, and a jug. Caterpillar juice and snake elixir were not in a vile, a vessel with the spider ointment was between the jar and the vessel with the beladonna extract; the jar is not with the spider ointment or the caterpillar juice; the glass was next to the jug and a vessel with spider ointment. In which vessel is each liquid stored? ( 35 pts.)



Answer: Spider ointment in a vile, caterpillar juice is in the jug, snake elixir is in the jar, beladonna extract in the glass.

3. A numeral palindrome is a magic 'symmetrical' number that remains the same when its digits are reversed (example: 12344321). The Warlock Mathematical Society offered a prize for the largest 2018-digit numeral prime palindrome by Halloween 2018. The prize was promised to be equal to the sum of all digits in the palindrome, in gold Kronas. How many gold Kronas does the Society need to prepare by Halloween to make sure they have enough? ( 40 pts.)



Answer: None, because the 2018-digit numeral prime palindrome doesn't exist. Therefore the Warlock Mathematical Society is not able to provide this prize.

Solution: Note that the 2018-digit palindrome  $N$  is divisible by 11 and cannot be a prime because for  $N = \underbrace{d_1 d_2 d_3 \dots \dots d_3 d_2 d_1}_{2018 \text{ digits}}$  the alternating sum of digits is  $d_1 - d_2 + d_3 - d_4 + \dots + d_{1009} -$

$d_{1009} + \dots - d_3 + d_2 - d_1 = 0$ . According to the rule of divisibility by 11 the number  $N$  is divisible by it. Note that the only one prime palindrome with the even number of digits does exist. It is 11