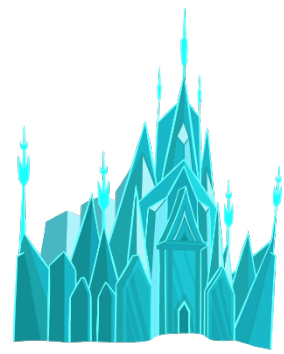
RSM-MetroWest, 5 Auburn St. Framingham, MA 01701 Tel: 508-283-1355

2017-2018 Puzzle Contests

Solutions for Contest #3

Grades 5up Puzzle Solutions:

1. Snow Queen's Castle has several Ice Towers where the cruel Snow Queen keeps her prisoners. Each Ice Tower (and there are more than one of them) has the same number of ice cells, and each ice cell holds the same number of prisoners. The number of ice cells in each Ice Tower is more than the number of prisoners in each cell, and the number of prisoners is more than the number of Ice Towers. How many ice cells are there in all Ice Towers of the Castle altogether, if there are 105 prisoners in all Towers? **(25 points)**



Answer: The total number of ice cells is 3 x 7 = 21

Solution: Let t denote the number of Ice Towers in the Castle, c the number of cells per Ice Tower, and p the number of prisoners per cell. Then $t \times c \times p = 105 = 3 \cdot 5 \cdot 7$. Note that the numbers $3, 5$ and 7 are prime. Since $1 < t < p < c$, $c = 7$

2. Each of the letters S, N, O and W in this multiplication stands for a different digit.

```
       S N O W
     × S N O W
    ----------
     * * * * S
   * * * * N
  * * * * O
* * * * W
----------
* * * * * * * *
```

What are the values of the letters of S N O W? **(35 points)**

Answer: $SNOW \rightarrow 6284\ or\ 6824$

Solution: Note that the last digit of $W \times W \times W$ is W itself, because $W \times W$ has the last digit S and $W \times S$ has the last digit W. Thus W has to be $4\ or\ 9$ (the other cases 0, 1, 5, 6 are excluded because they lead to $W = S$). 9 is excluded because then S = 1, which leads to 4 digits in the fourth row, instead of 5. So $W = 4, S = 6$. Then both $O\ and\ N$ have to be even ($W \times O$ has N as its last digit, while $W \times W$ has O as its last digit). Therefore, $N = 2,\ O = 8$ or $N = 8, O = 2$. Both cases satisfy the conditions of the problem. Finally, $SNOW = 6284\ or\ SNOW = 6824$.

3. Snow Queen challenged Gerda to a game of ice cubes.

The game of ice cubes is played as follows.
There is a heap of 2018 ice cubes. Players take turns taking ice cubes out of the heap. Each player can take 1, 2, or 3 ice cubes on each move. The player who takes the last ice cube wins.

Gerda's is allowed the first move. Help Gerda find a winning strategy that will help her win and free her little brother Kai, regardless of how the Snow Queen plays the game. **(40 points)**

Answer: The winning strategy of Gerda is to leave for his opponent the number of ice cubes which is divisible by 4 on each of her moves. So her first move is to take away two ice cubes leaving 2016 ice cubes in the heap. One each next move she should take the number of cubes equal to 4 minus the number of cubes Snow Queen takes on her last move, thus keeping the number of cubes remaining in the heap divisible by 4 every time.