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2016-2017 Puzzle Contests

Solutions for Contest #5

Grades 5 and up Puzzle Solutions:

1. The code of Alex's digital lock in the locker room is a two-digit number. Alex has forgotten the code and cannot unlock his locker after his soccer practice. However, he remembers that the sum of the digits for the code plus the product of the digits is the code itself. Explain how Alex can open his locker with the minimum of tries. **(25 pts)**

Solution: If a and b are the tens and ones digits of the code, respectively, then according to the problem, $a + b + a \cdot b = 10a + b$. By simplifying, $9a = a \cdot b$. Since $a \neq 0$ (a two-digit number!), then $b = 9$. But all possible values of a (a =1, 2,…, 9) still satisfy the problem, so Alex needs to try each of them (19, 29, … 99) until the lock opens—a maximum of 9 tries.

2. You won't find Math City on any map, but it does exist, and some mysterious, clever, and playful people live there. Once upon a time I went to see Ms. Zero, a very popular person in Math City. I asked the first person I met there: "Where does Ms. Zero live?" The puzzling reply was: "She lives in the middle of the long block on Boundary Street whose house numbers on that block total 767. Since Boundary St. faces Central Limit Park, it has houses on only one side." From this information I could figure out Ms. Zero's home address. Hope you can get it too. *Note that Math City has the strict numbering rule: each house has its own natural number that differs by 2 from its neighbors'*. **(35 pts)**

Solution: Note that $767 = 13 \ x \ 59$. No other product of natural numbers equals 767 because 13 and 59 are both prime. Because the total sum of all house numbers is the product of the number of houses and the average (middle) of the end house numbers, Ms. Zero's full address is 59 Boundary Street in Math City, and the number of houses in the block must be 13. (The street cannot have 59 houses because then some of the lower house numbers would be negative.)

3. Two players take turns moving an hour hand 2 or 3 hours forward. The initial position of the hour hand is at 12 o'clock. The goal of the game is to be the first to land exactly at 6 o'clock. The game continues around the clock until one player wins. Who wins this game if both players play the best they can--the player who moves first or the one who moves second? What is the winning strategy? **(40 pts)**

Answer: Let First (F) and Second (S) be the two players, with First starting. Step2 and Step3 are the hour hand moves of 2 and 3 hours forward respectively; AnyStep is either Step2 or Step3, and AddTo5 is an addition step that makes 5 with the previous step. Then *First can always win.* An idea of the winning strategy is to make 5 hours by the sum of two moves, e.g. First increases Second's move by his/her own move to sum to 5 (for example, Step2 has to be added to Step3 or Step3 has to be added to Step2). A sequence of winning moves is below:

(F) Step2-(S)Step3-(F)Step3-(S)AnyStep-(F)AddTo5-(S)AnyStep-(F)AddTo5 (6 o'clock !)