2017-2018 Puzzle Contests

Solutions for Contest #2

Parents and Grandparents Puzzle Solutions:

1. The coven of 45 witches came to the Halloween Party 2017 earing fashionable hats: 13 with red hats, 15 with green hats, and 17 with blue hats. After partaking in the party brews they started chasing each other.

If two witches wearing differently colored hats touched hands, they magically changed their hats color to the third one (for example: when witches wearng green and red hats touched hands, both changed their hats color to blue.) Could all witches end up changing their hats to the same color? Show the sequence of color changes, or prove that it is impossible. **(50 pts)**

Answer: that cannot happen. Here is the proof: It is easy to see that for any color-changing step, the pair-wise differences either increase by 3 or decrease by three or stay the same. It means they keep the same remainder after division by 3 for any color-changing step. If the same color could be possible then some two equal groups of witches should exist before last step. Then their numerical difference should be divisible by 3 for any color-changing step including an initial step. But none of possible initial differences (17-13=4, 17-15=2, 15-13=2) is divisible by 3. This contradictory proves our assertion.

2. Entrance to the the 2017 Halloween party requires a password. The password is a digital value of $\overline{TWOMIN}$ of the puzzles below

$$7 \times \overline{TWOMIN} = 6 \times \overline{MINTWO}$$

Here each letter is a decimal digit, the same letters are the same digits and different letters are different digits. Find the password and share it with us and your favourite witch so that she can join the Halloween Party **(30 pts.)**

Answer: TWO,MIN= 461,538.

Solution: Denote TWO as x and MIN as y. Then $7(1000x + y) = 6(1000y + x)$ or $6994 \cdot x = 5993 \cdot x$ or $538x = 461y$. Since 461 and 538 are co-prime numbers TWO= $x = 461$ and MIN= $y = 538$

3. A Path to Redemption which every zombie is searching for is said to be a 13-link closed broken line in which each link intersects exactly one of the other links (not at the vertex). A zombie that finds a path like that comes back to life. Can you help discover such a path? **(20 pts.)**

Answer: It is impossible to find such line
ProofL Assume it is possible to construct such 13-link closed broken line. Then intersecting links form the pairs. Hence the number of links has to be even. That contradicts the condition of the problem