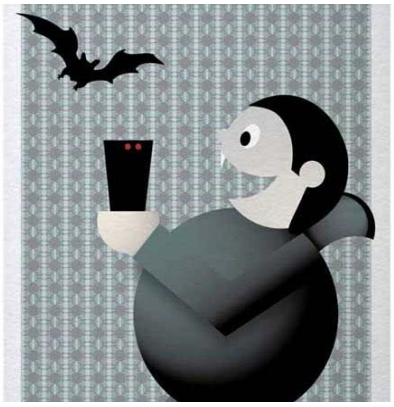
2018-19 Puzzle Contests

Solutions for Contest #2

Parents and Grandparents Puzzle Solutions:

1. Warlock, Vampire, Sorceress, Ghoul and Witch spent the evening before Halloween drinking in one of the five taverns around Sinville. Sinville Taverns did not boast variety – they sold either Black Toad Gin or Bloodroot Ale. And all the 5 taverns were on different streets. It so happened that Warlock, Vampire, Sorceress, Ghoul and Witch happened to wear different color cloaks on that day. Determine the name of each creature, what they drant, which street the tavern they at was located, and the color of their cloaks, if it is known that



- ✓ The witch did not wear purple. The Creature wearing purple and Ghoul drank Black Toad Gin.
- ✓ Grinwald did not drink the Black Toad Gin dressed in green. The vampire was at the tavern in Ghosts Way.
- ✓ Matilda was not at the Dragon Alley tavern. Gwendolyn was not a Sorceress.
- ✓ One of the five was drinking Black Toad Gin at a tavern in Hangman's row. Gwendolyn wore an orange cloak. Victor did not wear black.
- ✓ Grinwald the Warlock was not at the taven in Deadmans Road. The creature who drank Bloodrot Ale at a tavern in Black Crescent street wore a red cloak.
- ✓ The Ghoul was at the tavern in Dragon Alley. Raven, who was not a Sorceress, drank bloodroot Ale. **(25 pts.)**

Answer: See the answer in the table below

Name	Creature	Clak color	Gin/Ale	Street Name
Grinwald	Warlock	purple	Black Toad Gin	Hangman's row
Matilda	Sorceress	red	Bloodroot Ale	Black Cresscent
Raven	Vampire	black	Bloodroot Ale	Ghosts Way
Gwendolyn	Witch	orange	Bloodroot Ale	Deadmans Road
Victor	Ghoul	green	Black Toad Gin	Dragon Alley

2. Ghosts Grumzensel and Bruxdernstein were getting on in years. Grumzensel died in the 20th century in a brawl, while Bruxdernstein was poisoned in the 19th century . When they ran into each other on halloween night, they started reminiscing over the circumstances of their deaths. But their memories were getting dim, and neither of them could remember the exact date they were killded. Coincidentally however they both remembered that each was killed as many years ago as the the sum of digits of the year each was killed. How much longer has Bruxdernstein been dead than Grumszesel? **(35 pts.)**

Answer: Bruxdernstein has been dead 9 years longer than Grumzensel.
Solution: Denote $\overline{18xy} = 1800 + 10x + y$ the day of the year when Bruxdernstein died, $\overline{19uv} = 1900 + 10u + v$ the day of the year when Grumzensel died. Then $1800 + 10x + y + 1 + 8 + x + y$ is a year of their meeting, and so is $1900 + 10u + v + 1 + 9 + u + v$. Then $1809 + 11x + 2y = 1910 + 11u + 2v \rightarrow$
$11(x-u) + 2(y-v) = 101 \rightarrow 11(x-u-9) + 2(y-v-1) = 0$. The general solution of last equation is $\begin{cases} x-u = 9+2n \\ y-v = 1-11n \end{cases}$ where $n = 0, \pm 1, \pm 2, \ldots$. Since x, y, u and v are decimal digits then $n = 0$ and $\begin{cases} x-u = 9 \\ y-v = 1 \end{cases}$. Then the difference of the dates of death is $1900 + 10u + v - 1800 - 10x - y = 100 - 10(x-u) - (y-v) = 100 - 90 - 1 =$ 9 years i.e. Bruxdernstein has been dead exactly 9 years longer than Grumzensel. In addition note that although 1900 is the year of the 19th century and 2000 is the year of the 20th century they both cannot be solutions of our puzzle. Note also that a year of the meeting cannot be definitely determined.

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3. Professor Warlock, in his lecture at the School for Gifted and Talended Vampires, defined the set of all rational numbers such that $x \neq 0, x \neq 1$ *Cursed*, and denoted this set C. He further introduced a Ghastly function y = G(x) for which
   1. $x \in C$, $y \in R$ (R is the set of Real numbers)
   2. $G(x) + G\left(1 - \frac{1}{x}\right) = \log|x|$

   Find $G(2018)$ **(40 pts.)**

Answer: $\log\left(\frac{2018}{2017}\right)$

Solution: Consider the function $g(x) = 1 - \frac{1}{x}$ with $x \in C$. Then $g(x) \in C$. Note then that

$$g\left(g\left(g(x)\right)\right) = 1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}} = x$$

The given equation rewrites as $G(x) + G\left(g(x)\right) = \log|x|$. Substituting $x = g(y)$ and $x = g(g(z)$ gives the further equations that with initial equation gives the following system of three equations:

$$\begin{cases} G\left(g(y)\right) + G\left(g\left(g(y)\right)\right) = \log|g(y)| \\ G\left(g\left(g(z)\right)\right) + G\left(g\left(g\left(g(z)\right)\right)\right) = \log|g(g(x)| \\ G(x) + G(g(x)) = \log|x| \end{cases}$$

According to the note above $G\left(g\left(g\left(g(z)\right)\right)\right) = G(z)$ and our system transforms into the following

$$\begin{cases} G\left(g(y)\right) + G\left(g\left(g(y)\right)\right) = \log|g(y)| \\ G\left(g\left(g(z)\right)\right) + G(z) = \log|g(g(x)| \\ G(x) + G(g(x)) = \log|x| \end{cases}$$

Note that our functional relationships are true for any $x \in C, y \in C \; and \; z \in C$. Then put $x = y = z = 2018$ and get

$$\begin{cases} G\left(g(2018)\right) + G\left(g\left(2018\right)\right) = \log|g(2018)| \\ G\left(g\left(g(2018)\right)\right) + G(2018) = \log|g(g(2018)| \\ G(2018) + G(g(2018)) = \log|2018| \end{cases}$$

Eliminating the values $G\left(g(2018)\right) \; and \; G\left(g\left(g(2018)\right)\right)$ from the system gives

$$G(2018) = \frac{1}{2}\left(\log|2018| - \log|g(2018)| + \log|g(g(2018)|\right)$$

$$= \frac{1}{2}\left(\log 2018 - \log\left|\left(1 - \frac{1}{2018}\right)\right| + \log\left|1 - \frac{1}{1 - \frac{1}{2018}}\right|\right)$$

$$= \frac{1}{2}\left(\log 2018 - \log\frac{2017}{2018} + \log\frac{1}{2018}\right) = \log\frac{2018}{2017}$$