

# 2017-2018 Puzzle Contests

## Solutions for Contest #3



### Parents and Grandparents Puzzle Solutions:

**1. The Ancient Book** of Christmas Words of Wisdom states:

*A traveler lost in a blizzard upon open moorland on a Christmas Eve will safely reach his destination by Christmas if he utters the word "WIND" the smallest number times that is needed to make possible the identity*



$$WIND + WIND + \dots + WIND = AAAAAA$$

(a footnote in that book says that the same letters code the same digits, and different letters stand for different digits). What is that useful number? **(35 pts)**

**Answer:** The least number of items is 13.

**Proof:**  $WIND + WIND + \dots + WIND = n \cdot WIND = A \cdot 111,111 = A \cdot 3 \cdot 37 \cdot 7 \cdot 11 \cdot 13$ .  $n$  can not be equal to  $A$ ,  $3$ ,  $7$ , or  $11$  because the 4-digit number  $WIND$  doesn't exist then. Let's check  $n = 13$ . Then  $WIND = A \cdot 3 \cdot 37 \cdot 7 \cdot 11 = 8547 \cdot A$ . It is possible if  $A = 1$ ,  $WIND = 8547$  only. Note that the condition of the correspondence between digits and letters is satisfied,  $n$  is the smallest number, and there are no other solutions.

**2. It is rumored that the** 1,692,306<sup>th</sup> digit after decimal point of the number  $\{\sqrt{2026}\}^{1,000,000}$  will be the character of success in the year of 2026. What is this digit? Explain why.

( $\{x\}$  is a fractional part of real number  $x$ ; for example:  $\{1.999\} = 0.999$ ) **(40 pts)**

**Answer:** zero

**Proof:** Note that  $\sqrt{2026} = [\sqrt{2026}] + \{\sqrt{2026}\} = 45 + \{\sqrt{2026}\}$ . Hence  $\{\sqrt{2026}\} = \sqrt{2026} - 45 = \frac{1}{\sqrt{2026} + 45} < \frac{1}{90} < \frac{2}{100} = \frac{2}{10^2}$ . Then  $\{\sqrt{2026}\}^{1,000,000} < \left(\frac{2}{10^2}\right)^{1,000,000} = \frac{2^{1,000,000}}{10^{2,000,000}} = \frac{(2^{13})^{\frac{1,000,000}{13}}}{10^{2,000,000}} < \frac{10^{\frac{4,000,000}{13}}}{10^{2,000,000}} < \frac{10^{307,693}}{10^{2,000,000}} = \frac{1}{10^{1,692,307}}$ . Hence at least 1,692,306 digits after decimal point of the number  $\{\sqrt{2026}\}$  are zeros.

In this solution  $\{x\}$  is a fractional part of real number  $x$ ;  $[x]$  is the greatest integer part of real number  $x$ ; for example  $\{1,23456\} = 0.23456$ ;  $[1,23456] = 1$ .

**3. In the periodic decimal representation** of  $\frac{1}{700}$ , all digits starting from the 2018<sup>th</sup> after the decimal point were shifted 1 position to the left, replacing the 2017<sup>th</sup> digit. All digits to the left of the 2017<sup>th</sup> after the decimal remained unchanged. Is the new number greater or smaller than  $\frac{1}{700}$ ? Why or why not? (25 pts)

Answer: The resulting number is larger than  $\frac{1}{700}$ .

Solution: The number  $\frac{1}{700}$  can be represented as an infinite periodic decimal

$\frac{1}{700} = 0.00142857142857 \dots 142857 \dots$  Its period starts from the third digit after the decimal point (the first two digits are zeros) and consist of 6 digits. Since  $2017 - 2 = 2015$  gives the remainder 5 after division by 6, the 2017<sup>th</sup> digit of the initial number is 5. After removing it, and shifting the digits to the right one position to the left, 7 replaces 5 in the 2017<sup>th</sup> position. Therefore the new number is larger than the initial number.