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2016-2017 Puzzle Contests

Solutions for Contest #5

Parents and Grandparents Puzzle Solutions:

1. Remove one of the factorials from the product 1! · 2! · 3! ·…· 2019! · 2020! so that the product of the remaining factors forms a perfect square.

(! is the product of all natural numbers from 1 to N ; $N! = 1\cdot 2\cdot 3\cdot \ldots \cdot N$) **(25 pts)**

Factorial Formula

Factorial formula by **Product**

$$n! = \prod_{k=1}^{n} k$$

Factorial Formula by **Recurrence Relation**

$$n! = \begin{cases} 1 & if\ n = 0, \\ (n-1)! * n & if\ n > 0 \end{cases}$$

Answer: 1010! has to be deleted.

Proof: $1!\cdot 2!\cdot 3!\cdot\ldots\cdot 2019!\cdot 2020!$

$= 2(1!)^2\cdot 4\cdot (3!)^2\cdot\ldots\cdot 2018\cdot (2017!)^2\cdot 2020\cdot (2019!)^2 =$

$= 2^{1010}\cdot (1010!)\cdot (1!)^2\cdot (3!)^2\cdot (5!)^2\cdot\ldots\cdot (2019!)^2 = (1010!)\cdot (2^{505}\cdot 1!\cdot 3!\cdot 5!\cdot\ldots 2019!)^2$.

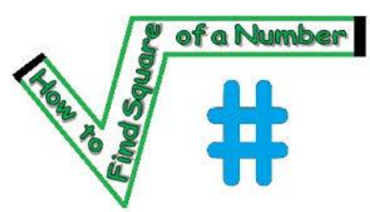
Hence (1010!) has to be deleted for getting a perfect square.

2. Every letter of the Cryptogram $\frac{\overline{AHHAAH}}{\overline{JOKE}} = \overline{HA}$ represents a non-zero decimal digit uniquely. What is $\overline{JOKE}$? **(35 pts)**

Answer: $\overline{JOKE}$=5169.

Solution: $\overline{JOKE} = \frac{\overline{AHHAAH}}{\overline{HA}} = 100 + \frac{\overline{AH}(10001)}{\overline{HA}} = 100 + \frac{\overline{AH}\cdot 73\cdot 137}{\overline{HA}}$. Then $\overline{HA} = 73$. Hence $\overline{JOKE} = \frac{377337}{73} = 5169$

3. What is the sum of the first 2017 digits after the decimal point in the number $(\sqrt{50}+7)^{2017}$? Why? **(40 pts)**



Answer: The sum of the digits is zero.

Proof: Note that $(\sqrt{50}+7)^{2017} - (\sqrt{50}-7)^{2017}$ is a natural number, Denote it N. Then $N < (\sqrt{50}+7)^{2017} = N + (\sqrt{50}-7)^{2017} < N + \frac{1}{(\sqrt{50}+7)^{2017}} < N + \frac{1}{(7+7)^{2017}} < N + \frac{1}{14^{2017}} < N + \frac{1}{10^{2017}}$. This means that all decimal places after the decimal point and at least until the 2017th are filled by 0 and the mentioned sum is 0 then.